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Analysis of Stress Intensity Factors of a Planar Rectangular Interfacial Crack in Three Dimensional Bimaterials

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Abstract. In this study, a rectangular interfacial crack in three dimensional bimaterials is analyzed. First, the problem is formulated as a system of singular integral equations on the basis of the body force method. In the numerical analysis, unknown body force densities are approximated by the products of the fundamental density functions and power series, where the fundamental density functions are chosen to express a two-dimensional interface crack exactly. The calculation shows that the present method gives smooth variations of stress intensity factor along the crack front for various aspect ratios. The present method gives rapidly converging numerical results and highly satisfied boundary conditions throughout the crack boundary. It is found that the stress intensity factors K_1 and K_2 are determined by bimaterials constant ε alone, independent of elastic modulus ratio and Poisson's ratio.

Introduction

With the rapidly increasing use of composite materials and adhesive, much attention has been paid to the interfacial crack by researchers. The fracture of composites and bonded dissimilar materials is induced mainly form the interfacial region. It is useful to design and manufacture composite materials for which we know the fracture behavior of their interface. Hence, problem of interface crack in dissimilar materials is very important. However, most of these works are on two-dimensional cases. Due to the mathematical difficulties, there are not any analytical methods for three-dimensional crack problems. However several numerical methods are available. The numerical solutions are discussed in the analysis of a penny-shaped crack between two dissimilar materials [1-5]. Noda et al. [6] evaluated the stress intensity factors of an axi-symmetric interface cracks under torsion and tension by a body force method. Chen [7] deals with linear elastic fracture problems for a planar crack on an interface between two dissimilar elastic half-space solids bonded together. Noda, Miyoshi, et al [8] studied mixed modes stress intensity factors of an inclined semi-elliptical surface crack by a body force method, in which the unknown body force densities were approximated by the products of fundamental density functions and polynomials. This numerical method was applied by Wang and Noda [10] to investigate the stress intensity factors of a 3D rectangular crack using body force method.

In the previous papers [9], the hypersingular intergro-differential equations for the planar interface crack were indicated. However, the oscillation singularity as well as overlapping of crack surfaces near the crack tip makes it mach more difficult exactly to solve the equations compared with the cases of the ordinary cracks. In this paper, the numerical solutions are considered for the planar rectangular interface crack on the basis of the equations. The fundamental density functions are chosen to express a two-dimensional interface crack exactly. Then, it will be shown that the smooth variations of stress intensity factor along the crack front are highly satisfactory boundary conditions throughout the crack surface.

Singular integral equations for a planar interfacial crack

Consider two dissimilar elastic half-spaces bonded together along the *x–y* plane.(see Fig. 1). Suppose that the upper half-space is occupied by an elastic medium with constants (μ_1, ν_1) and the lower half-space by an elastic medium with constants(μ_2, ν_2) , where μ is the shear modulus and ν is the Poisson's ratio. The crack is assumed to be located at the bimateral interface.

Singular integral equations for three dimensional crack problem on bimateral interface in Fig. 1 were derived by Chen [7] as shown in Eq.1(a)-1(e).

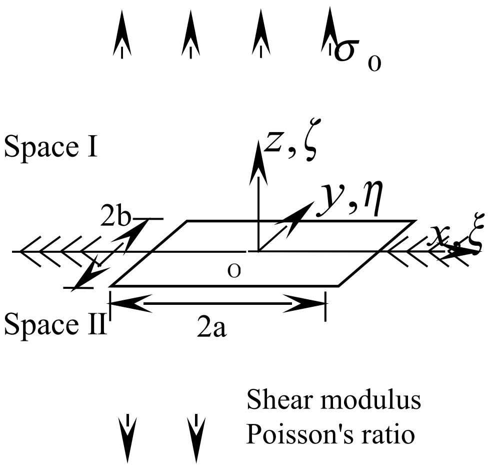

Fig. 1 Problem configuration

$$\mu_1(\Lambda_2 - \Lambda_1)\frac{\partial \Delta u_z(x,y)}{\partial x} + \mu_1 \frac{(2\Lambda - \Lambda_1 - \Lambda_2)}{2\pi}\int_s \frac{1}{r^3}\Delta u_x(\xi,\eta)dS(\xi,\eta)$$
$$+3\mu_1\frac{(\Lambda_1 + \Lambda_2 - \Lambda)}{2\pi}\left\{\int_s \frac{(x-\xi)^2}{r^5}\Delta u_x(\xi,\eta)dS(\xi,\eta) + \int_s \frac{(x-\xi)(y-\eta)}{r^5}\Delta u_y(\xi,\eta)dS(\xi,\eta)\right\} = 0 \quad (1a)$$

$$\mu_1(\Lambda_2 - \Lambda_1)\frac{\partial \Delta u_z(x,y)}{\partial y} + \mu_1 \frac{(2\Lambda - \Lambda_1 - \Lambda_2)}{2\pi}\int_s \frac{1}{r^3}\Delta u_y(\xi,\eta)dS(\xi,\eta)$$
$$+3\mu_1\frac{(\Lambda_1 + \Lambda_2 - \Lambda)}{2\pi}\left\{\int_s \frac{(x-\xi)(y-\eta)}{r^5}\Delta u_x(\xi,\eta)dS(\xi,\eta) + \int_s \frac{(y-\eta)^2}{r^5}\Delta u_y(\xi,\eta)dS(\xi,\eta)\right\} = 0 \quad (1b)$$

$$\mu_1(\Lambda_1 - \Lambda_2)\left(\frac{\partial \Delta u_x(x,y)}{\partial x} + \frac{\partial \Delta u_y(x,y)}{\partial y}\right) + \mu_1\frac{(\Lambda_1 + \Lambda_2)}{2\pi}\int_s \frac{1}{r^3}\Delta u_z(\xi,\eta)dS(\xi,\eta) = -p_z(x,y) \quad (1c)$$

$$\Lambda = \frac{\mu_2}{\mu_1 + \mu_2}, \quad \Lambda_1 = \frac{\mu_2}{\mu_1 + \kappa_1\mu_2} \quad \Lambda_2 = \frac{\mu_2}{\mu_2 + \kappa_2\mu_1}$$
$$\kappa_1 = 3 - 4\nu_1, \quad \kappa_2 = 3 - 4\nu_2 \quad r^2 = (x-\xi)^2 + (y-\eta)^2 \quad (1d)$$

$$\Delta u_i(x,y) = u_i(x,y,0^+) - u_i(x,y,0^-), (i = x,y,z) \quad (1e)$$

In Eq.(1a)-(1e),unknown functions are crack opening displacements, in other words, displacement discontinuities$\Delta u_x, \Delta u_y, \Delta u_z$ defined in Eq.(1e). Here, p_z are stresses σ_z at infinity. Here the integration should be interpreted in the sense of a finite part integral in the region *S*.

Numerical solutions of singular integral equations

In the present analysis, the trigonometric function and polynomials have been used to approximate the unknown functions as a continuous function. First, we put

$$\Delta u_i(\xi,\eta) = w_i(\xi,\eta)F_i(\xi,\eta),\ i = x,y,z \tag{2}$$

For the three dimensional crack problem, the fundamental density function are expressed as the follows

$$\left.\begin{aligned}
w_x(\xi,\eta) &= \sum_{l=1}^{2} \frac{1+k_l}{4\mu_l \cosh\pi\varepsilon}\sqrt{a^2-\xi^2}\sqrt{b^2-\eta^2}\sin\left(\varepsilon\ln\left(\frac{a-\xi}{a+\xi}\right)\right)\\
w_y(\xi,\eta) &= \sum_{l=1}^{2} \frac{1+k_l}{4\mu_l \cosh\pi\varepsilon}\sqrt{a^2-\xi^2}\sqrt{b^2-\eta^2}\sin\left(\varepsilon\ln\left(\frac{b-\eta}{b+\eta}\right)\right)\\
w_z(\xi,\eta) &= \sum_{l=1}^{2} \frac{1+k_l}{4\mu_l \cosh\pi\varepsilon}\sqrt{a^2-\xi^2}\sqrt{b^2-\eta^2}\cos\left(\varepsilon\ln\left(\frac{a-\xi}{a+\xi}\right)\right)\cos\left(\varepsilon\ln\left(\frac{b-\eta}{b+\eta}\right)\right)
\end{aligned}\right\} \tag{3}$$

Here,

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{\mu_2\kappa_1+\mu_1}{\mu_1\kappa_2+\mu_2}\right) \tag{4}$$

The following expressions can be applied, where the unknowns are coefficients of the polynomials $\alpha_i, \beta_i, \gamma_i$

$$F_x(\xi,\eta) = \sum_{i=0}^{l-1}\alpha_i G_i(\xi,\eta)\,,\ F_y(\xi,\eta) = \sum_{i=0}^{l-1}\beta_i G_i(\xi,\eta)\,,\ F_z(\xi,\eta) = \sum_{i=0}^{l-1}\gamma_i G_i(\xi,\eta)$$

$$l = (n+1)(n+1)$$

$$G_0(\xi,\eta) = 1,\ G_1(\xi,\eta) = \eta,\ G_2(\xi,\eta) = \eta^2 \cdots, G_{n+1}(\xi,\eta) = \xi,\ G_{n+2}(\xi,\eta) = \xi\eta \cdots, G_{l-1}(\xi,\eta) = \xi^m\eta^n$$

Numerical results and conclusions

Consider a rectangular interfacial crack in three dimensional infinite elastic solid under a uniform tension load $p_z = 1$.In demonstrating the numerical results of stress intensity factor (SIF), the following dimensionless factors F_1 ,F_2 and F_3 will be used. Several examples are given in the Table1. The results show that the present solution has good convergence, and highly satisfied boundary conditions. In Table1, it is seen that the stress intensity factor at the center of the crack front for the case of $a/b \ge 8$ is very closed to that of two-dimension case. It is found that the stress intensity factors K_1 and K_2 are determined by bimaterials constant ε alone, independent of elastic modulus ratio and Poisson's ratio. Since the values of K_3 is far smaller than K_1 and K_2, are not given in this paper. The dimensionless stress intensity factor F_1 and F_2 for a/b=1, 2, 4, 8 at the point (0, ± 1) are given by Table 2. It is shown that the values of F_1 are decrease with increasing parameter ε , but the values of F_2 are increase with increasing parameter ε .

$$F_1 + iF_2 = \frac{K_1(x,y)\big|_{x=x,y=\pm b} + iK_2(x,y)\big|_{x=x,y=\pm b}}{\sigma_{33}^{\infty}\sqrt{\pi b}} = \sqrt{a^2-x^2}\left(\cos\left(\varepsilon\ln\left(\frac{a-x}{a+x}\right)\right)F_z(x,y)\big|_{x=x,y=\pm b} + 2i\varepsilon F_y(x,y)\big|_{x=x,y=\pm b}\right)$$

$$F_3 = \frac{K_3(x,y)\big|_{x=x,y=\pm b}}{\sigma_{33}^{\infty}\sqrt{\pi b}} = \sum_{l=1}^{2}\frac{1+\kappa_l}{4\mu_l\cosh\pi\varepsilon}\frac{1}{(1/G_1+1/G_2)}\sqrt{a^2-x^2}\sin\left(\varepsilon\ln\left(\frac{a-x}{a+x}\right)\right)F_x\big|_{x=x,y=\pm b}$$

Table1 Stress intensity factor F_1 and F_2 at x=0, y=b

	ν_1	ν_2	ε =0.02				ε =0.04			
			a/b=1	a/b=2	a/b=4	a/b=8	a/b=1	a/b=2	a/b=4	a/b=8
F_1	0.3	0.3	0.7528	0.9052	0.9760	0.9947	0.7509	0.9038	0.9750	0.9938
	0	0.5	0.7527	0.9052	0.9760	0.9947	0.7508	0.9037	0.9749	0.9938
	0	0	0.7528	0.9053	0.9761	0.9947	0.7511	0.9040	0.9751	0.9938
F_2	0.3	0.3	0.0274	0.0352	0.0388	0.0396	0.0542	0.0696	0.0768	0.0786
	0	0.5	0.0271	0.0349	0.0387	0.0396	0.0537	0.0692	0.0766	0.0786
	0	0	0.0277	0.0355	0.0389	0.0396	0.0549	0.0704	0.0770	0.0786

Table 2 Dimensionless stress intensity factor F_1 and F_2 at the point (0,1)

	F_1				F_2			
	a/b=1	a/b=2	a/b=4	a/b=8	a/b=1	a/b=2	a/b=4	a/b=8
ε =0.02	0.7528	0.9052	0.9760	0.9947	0.0274	0.0352	0.0388	0.0397
ε =0.04	0.7509	0.9038	0.9750	0.9938	0.0542	0.0696	0.0768	0.0786
ε =0.06	0.7478	0.9013	0.9730	0.9920	0.0799	0.1027	0.1134	0.1160
ε =0.08	0.7433	0.8975	0.9699	0.9891	0.1040	0.1338	0.1479	0.1515
ε =0.10	0.7373	0.8921	0.9654	0.9848	0.1263	0.1627	0.1801	0.1845

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